Chaotic Composition: Generating Unique Music through Nonlinear Systems

Devyansh Khanna

October 13, 2025

Abstract

Chaos theory describes systems that are fully deterministic and predictable, yet often appear random. These systems are highly sensitive to initial conditions and can produce drastically different results due to slight initial changes. This paper explores how chaos theory can be used in the creation of music, using nonlinear systems such as logistic maps and Lorenz attractors to determine musical factors such as rhythm and pitch. This has applications in both music and mathematics, as chaotic algorithms used for music generation can lead to the creation of unique forms, and the use of music provides a framework for further exploration into chaos theory.

1 Introduction

In chaos theory, systems are deterministic, i.e., governed by a defined rule that provides results systematically. However, these systems can change drastically when the initial conditions are slightly altered. Even minimal alterations can lead to seemingly "random" changes. This behavior arises through nonlinear systems, i.e., systems that do not adhere to the superposition principle:

$$f(ax_1 + bx_2) \neq af(x_1) + bf(x_2) \tag{1}$$

(Condition for a Linear System)

Musical structures often share similarities with chaotic systems—having deterministic frameworks but producing outputs that appear unpredictable. Chaos theory often results in the formation of statistical fractals, i.e., fractals that follow specific time-series patterns of the form

$$S(f) = \frac{1}{f} \tag{2}$$

This exact relation is used to describe the statistical power series followed by most music, which is also termed "pink noise" [6]. Music and chaotic systems thus hold a statistical relationship, making their combination both feasible and insightful.

Therefore, it follows that chaos theory can be used in music creation to yield unique results. We will now attempt to form an algorithm for the generation of chaotic music.

2 Theoretical Background

2.1 The Logistic Map

The logistic map is one of the most well-known examples of a chaotic system, recognized for its distinctive shape. It is a simple one-dimensional chaotic system defined by:

$$x_{n+1} = rx_n(1 - x_n) \tag{3}$$

When r exceeds approximately 3.57, the system becomes chaotic. Although it is a simple relation, the sequences produced are highly chaotic and unpredictable if the original parameters are unknown. For values under $r = 3$, the series settles to a stable value. When r exceeds 3, this stable state bifurcates into two stable states, and this bifurcation continues until a chaotic regime is achieved at $r = 3.57$. If we plot the stable state values against the values of r, we obtain the characteristic graph as seen in Figure 1.

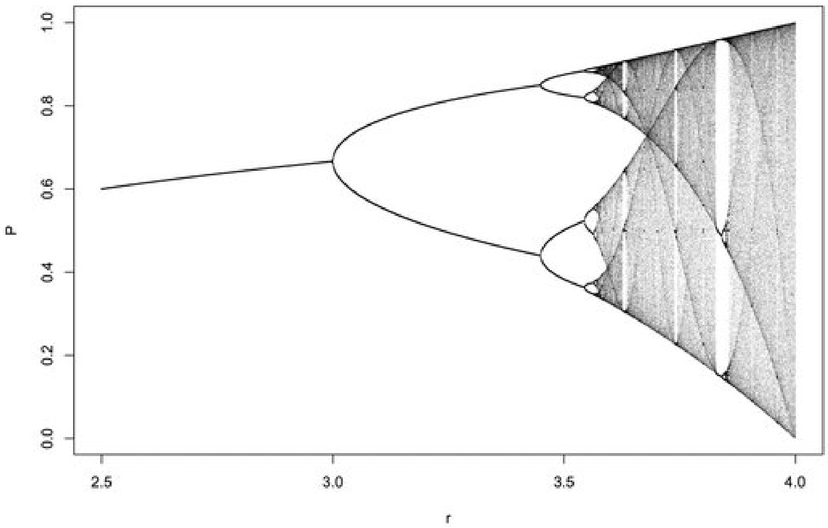

Figure 1: The Logistic Map.

2.2 The Lorenz Attractor

Another well-known example of a chaotic system is the Lorenz attractor [2], a type of "strange attractor" that describes points in three-dimensional space. The mapping of these points reveals a pattern, but no point ever repeats. The Lorenz attractor is given by:

$$\dot{x} = \sigma(y - x), \tag{4}$$

$$\dot{y} = x(\rho - z) - y, \tag{5}$$

$$\dot{z} = xy - \beta z \tag{6}$$

For typical parameters $(\sigma, \rho, \beta) = (10, 28, 8/3)$, the trajectory traces a butterfly-like pattern in phase space, demonstrating deterministic chaos. This is often used to analogize chaos theory through the "butterfly effect," representing deterministic yet unpredictable consequences.

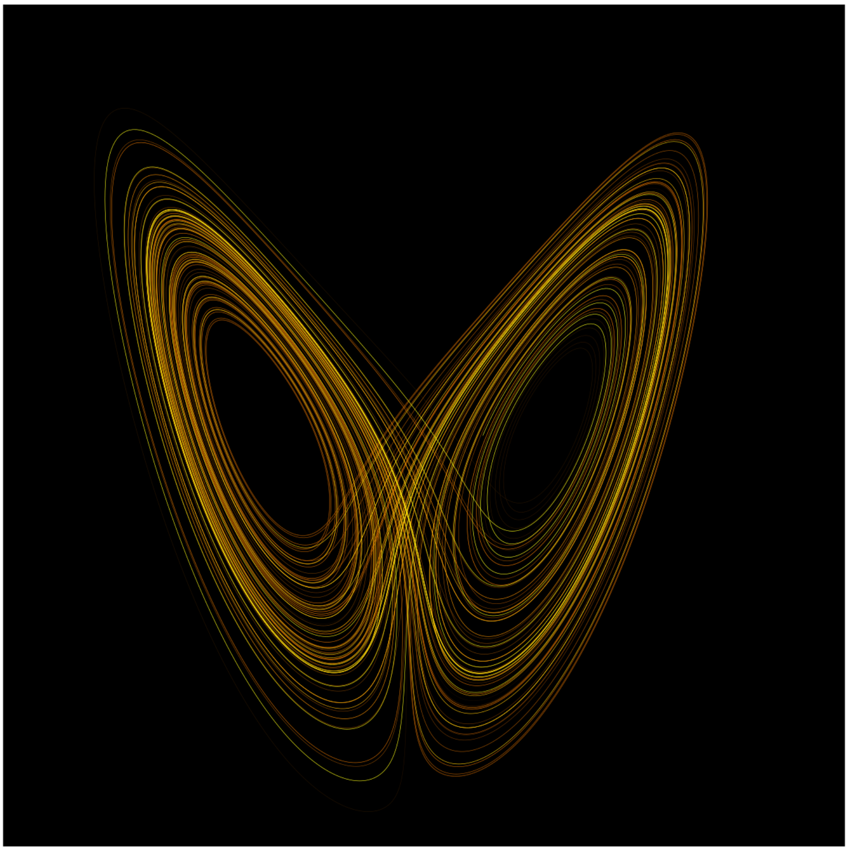

Figure 2: Lorenz attractor phase plot illustrating chaotic dynamics.

3 Methodology

3.1 Data Generation

For basic data generation, the logistic map is used to create multiple series values. In our case, we take $r = 3.8$ and $x_0 = 0.27$. Using these parameters, we obtain a chaotic pattern where no two values repeat, and no simple pattern can be discerned despite the deterministic nature of the series.

By iterating the given values multiple times, we obtain a chaotic series with numerous non-repeating points. Changing the value of x_0 by even a small amount produces a completely new series that does not resemble the earlier one.

Thus, we can create a data generator using the logistic map. This principle is used in several fields, such as pseudo-random number generation [5]. These numbers are not truly random since they are generated algorithmically; however, they are functionally equivalent for most applications. We will use our data generator to create music.

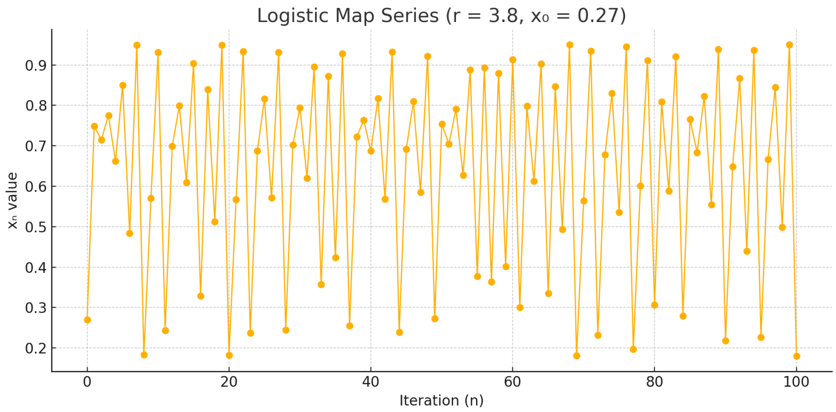

Figure 3: The logistic map for $r = 3.8$ and $x_0 = 0.27$.

3.2 Mapping Chaotic Data to Music

The logistic map produces a dataset containing values in the interval [0,1]. These values can be mapped to musical parameters to generate rough musical patterns:

- **Pitch:** Values from [0,1] are mapped to the MIDI range (60–84) to generate pitches.
- **Duration:** Values are quantized to rhythmic categories (quarter, eighth, or sixteenth notes).
- **Dynamics:** Values from [0,1] are mapped to MIDI velocity (60–120).

Using these mappings, we can generate unique musical notes that can then be arranged into compositions.

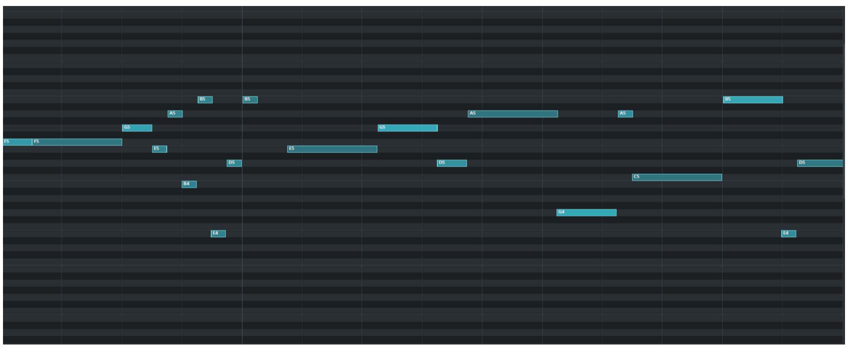

Figure 4: The MIDI sequence generated using a Logistic Map.

4 Results and Analysis

As seen in Figure 4, the generated sequence appears somewhat structured but retains a largely chaotic nature. The note lengths do not follow any fixed pattern, and while the notes remain close in pitch, they are unpredictable. This specific sequence was quantized to the C Major scale, which is why there are no sharp or flat pitches.

Upon listening, the sequence does not sound musically coherent or traditionally pleasing. However, closer inspection reveals micro-patterns, such as repeating note lengths of 0.25, which recur systematically [4].

Figure 5: Repeating patterns in chaotic music.

Figure 5 shows such a pattern repeated three times in the course of the sequence. Another example of repetition is shown in Figure 6.

Figure 6: A consecutive repeating pattern of notes.

This demonstrates that patterns can still be found in music generated through chaos theory, and that through human manipulation, the chaotic generation system can form unique musical ideas useful for composition and sound design [1]. Although initial conditions result in a specific sequence, any alteration will form a completely new and distinct set of notes.

5 Discussion

5.1 Advantages

The chaotic model for music generation presents a novel and useful way to produce unique patterns, offering exciting opportunities for composers and artists.

- **Unique Patterns:** The distinct musical patterns generated by the model are non-replicable and vary drastically depending on initial conditions. This allows for functionally infinite generations.

- **Fractality:** The patterns are fractal in nature, as the logistic map exhibits recursive repetition similar to the Mandelbrot set [3]. This mirrors the fractal structures statistically found in natural music ($S(f) = 1/f$).

- **Simplicity:** The model is computationally light, requiring only a short equation and two parameters, making it accessible and efficient for creative exploration.

5.2 Limitations

Despite its positive aspects, this model also suffers from drawbacks that can limit its use in real music generation.

- **Musical Coherence:** The musical structure generated by mapping chaotic values directly to MIDI output often lacks coherent harmonic or melodic flow. Refinement and post-processing are required for musically meaningful results.

- **Subjectivity:** The resulting melody, while statistically structured, remains open to interpretation. Different listeners may perceive the same sequence in entirely different ways.

6 Conclusion

This paper demonstrates the use of chaos theory in generating musically meaningful material. The self-similar and deterministic nature of chaos resembles natural musical behavior. This opens the door for further work on the role of chaos theory in algorithmic composition. The use of other chaotic systems, such as the Lorenz attractor, could further enrich this field. However, challenges remain, including control over parameters and improving the musical merit of the generated sequences. Overall, the chaotic music generator presents a promising intersection between mathematics and creativity, illustrating how deterministic equations can produce expressive, human-like variation.

References

[1] Li Hsu. Chaotic music: The structure of nonlinearity. *Chaos, Solitons & Fractals*, 13 (8):1499–1505, 2001. doi: 10.1016/S0960-0779(00)00126-8.

[2] Edward N. Lorenz. Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences*, 20(2):130–141, 1963. doi: 10.1175/1520-0469(1963)020⟨0130:DNF⟩2.0.CO;2.

[3] Benoit B. Mandelbrot. *The Fractal Geometry of Nature*. W. H. Freeman and Company, New York, 1983.

[4] Efthimios Manousakis. Musical composition based on deterministic chaos. *Leonardo Music Journal*, 6(1):57–61, 1996. doi: 10.2307/1513186.

[5] Robert M. May. Simple mathematical models with very complicated dynamics. *Nature*, 261:459–467, 1976. doi: 10.1038/261459a0.

[6] Richard F. Voss and John Clarke. '1/f noise' in music and speech. *Nature*, 258: 317–318, 1975. doi: 10.1038/258317a0.